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US to the opposite umbilic U'. Then, by reason of the symmetry of the surface, we shall have the angle $SU'U$ equal to SU_1U', and supplemental to SU_1U. Consequently the tangents of the halves of SUU_1 and $SU'U_1$ are to each other in a constant ratio. From this it appears that the principal ellipse passing through the umbilics is a common asymptot to all the geodetic lines which pass through either umbilic.

" Though the umbilical geodetics are thus shown to be infinite spires, it is not true that all the geodetics on the surface of an ellipsoid are of the same nature. Besides the geodetics which coincide with the principal sections of the surface, there are others among them which are closed curves. An example will make this evident.

" A circular disc, with a regular figure of an even number of sides inscribed in it, may be regarded as an infinitely flattened spheroid of revolution, with a continuous geodetic line traced upon its surface. In fact a closed cord, carried along in the direction of the sides of the regular figure, and passing over at each angle to the opposite side of the disc, would be kept stretched round it.

" I hope to be able, before long, to communicate to the Academy a series of remarkable results respecting the comparison of similar geodetic arcs, at which I have arrived by means of the theorems stated in the beginning of the present note. I expect also, by the translation of these geometrical theorems into analytical language, to obtain some new relations between the integrals, to the consideration of which we are led in the rectification of the geodetic lines and lines of curvature of a surface of the second order."

Rev. William Roberts communicated an analytic proof of the theorem stated by Mr. Graves, and made some observations on different applications of the formula of M. Liouville, on which the proof depends.

" The theorem which Mr. Graves has announced results in a very simple manner from a formula given for the first time, I believe, by Mr. Haedekampe, of Hamm, in the twenty-fifth volume of Crelle's Journal, page 180. In a memoir published in the twelfth volume of the *Journal de Mathematiques*, M. Liouville has demonstrated the same formula from geometrical considerations, and has attached a very elegant and precise signification to it. As the theorem in question, regarded under this point of view, is the natural extension to the case of three dimensions of the property (discovered by Mr. Graves), relating to the excess of the sum of the tangents drawn to an ellipse from a point upon a confocal ellipse over the included arc, it will be well to show how this latter proposition may be established by the method to which I allude. The generalization will be seen to follow without any difficulty.

" Let us adopt the notation of elliptic co-ordinates. The differential of the length of a right line tangent to an ellipse defined by the equation $\mu = a$, will be

$$d\mu\sqrt{\left(\frac{\mu^2 - a^2}{\mu^2 - b^2}\right)} + d\nu\sqrt{\left(\frac{a^2 - \nu^2}{b^2 - \nu^2}\right)},$$

where $2b$ is the constant distance between the foci. Now, if we remember that the second term is precisely the differential of an arc of this ellipse, and if we fix the origin of the ν's at a point (P) on the ellipse, it is clear that the above expression will be equally the differential of the mixtilineal line, composed of an elliptic arc, one of whose extremities (P) is fixed, and of the right line tangent on the other extremity. Let us now consider a pair of tangents drawn from a point O to an ellipse, and let T, T' be the points of contact, and P, Q two fixed points on the ellipse. Then the differential of the mixtilineal line $OT + TP$ will be

$$\sqrt{\left(\frac{\mu^2 - a^2}{\mu^2 - b^2}\right)} + d\nu\sqrt{\left(\frac{a^2 - \nu^2}{b^2 - \nu^2}\right)},$$

and that of the mixtilineal line $O\,T' + T'Q$ will be

$$d\mu\sqrt{\left(\frac{\mu^2 - a^2}{\mu^2 - b^2}\right)} - d\nu\sqrt{\left(\frac{a^2 - \nu^2}{b^2 - \nu^2}\right)}.$$

Hence, $O\,T + O\,T' + (TP + T'Q)$ will be constant, if $d\mu = 0$, that is to say, if O describe an ellipse confocal with the given one.

" We may now pass to the consideration of the analogous theorem for the dimensions. The differential of a right line, common tangent to two confocal surfaces of the second degree, is in elliptic co-ordinates ρ, μ, ν,

$$d\rho\sqrt{\left\{\frac{(\rho^2 - a^2)(\rho^2 - \beta^2)}{(\rho^2 - b^2)(\rho^2 - c^2)}\right\}} \pm d\mu\sqrt{\left\{\frac{(a^2 - \mu^2)(\mu^2 - \beta^2)}{(\mu^2 - b^2)(c^2 - \mu^2)}\right\}}$$
$$\pm\, d\nu\sqrt{\left\{\frac{(a^2 - \nu^2)(\beta^2 - \nu^2)}{(b^2 - \nu^2)(c^2 - \nu^2)}\right\}} \qquad (a)$$

b, c being the well-known constants in this system of co-ordinates, and a, β, the parameters which determine the two surfaces above mentioned. Let one of them be defined by the equation $\rho = a$, and the sum of the second and third terms, in the foregoing expression, will be the differential of a geodesic line traced upon this surface. Hence, the expression (a) may be regarded as the differential of the mixtilineal line, composed of a geodesic line (G) upon the surface $\rho = a$, counted from a fixed point, and of the linear tangent to it at its variable extremity. And if we consider the geodesic line (G') upon the same surface for which the coefficients of $d\mu$ and $d\nu$ are of opposite signs from those in the case of the line (G), we may easily see that if the sum of two arcs of the lines G and G', counted from fixed points, together with the lengths of the tangents applied to them at their variable extremities (which obviously intersect), be constant, the locus of their intersection will lie in a surface having for equation $d\rho = 0$, that is to say, a surface confocal with the given one. The same expression (a) leads, with equal facility, to a theorem of M. Chasles respecting lines of curvature. In order to obtain this theorem it will be necessary to consider one term in the formula, which will be the differential of an arc of a line of curvature; just as, by considering two conjointly, we were led to the expression for the length of a geodesic arc.

"The formula of M. Liouville, which I have employed, admits of being interpreted in several ways. For instance, analytically speaking, it gives us Abel's theorem respecting the comparison of ultra-elliptic functions of the first class and second species; and, regarded under this point of view, it furnishes us with the solution of the following problem:

"'Being given three arcs of a line of curvature on a surface of the second degree, to determine two others dependent upon them algebraically, so that the sum of the five arcs, taken with their proper signs, may be equal to a right line.'

"In conclusion, I beg to disclaim any originality in the foregoing communication. Everything which I have advanced on the subject is implicitly contained in the very elaborate memoir of my friend M. Liouville."

FEBRUARY 26TH, 1849.

REV. HUMPHREY LLOYD, D. D., PRESIDENT,
in the Chair.

On the recommendation of the Council,

IT WAS RESOLVED,—That £100 be placed at the disposal of the Secretary of the Academy, for the purchase of Irish MSS. at the Stowe sale.

The President communicated some facts respecting the remarkable atmospheric wave which passed over Dublin in the course of the present month, together with a notice of the more